RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2016

FIRST YEAR [BATCH 2016-19]

PHYSICS [Honours]

Paper : I

Date : 12/12/2016

Time : 11 am – 3 pm

Full Marks : 100

[Use a separate Answer Book for each Unit]

Unit – I

(***Answer any three questions***) [3×10]

1. a) Write the sum S = A + B, Difference D = A – B, and products P = AB, Q = BA of the matrices

$$A=\begin{pmatrix}3 & 1 & -2\\ 4 & -2 & 3\\ -2 & 1 & -1\end{pmatrix},\ B=\begin{pmatrix}2 & 0 & -1\\ -4 & 1 & 2\\ 1 & -1 & 0\end{pmatrix}$$ [1+1+2+2]

b) Let A be a finite dimensional matrix with real entries such that $AA^T=1$, where A^T denotes the transpose of A. Show that $A^TA=1$. [2]

c) Show that a Hermitian matrix remains Hermitian under a unitary transformation. [2]

2. a) Find the unit tangent vector to any point on the curve $x=t^2+1,\ y=4t-3,\ z=2t^2-6t$. Determine the unit tangent at the point where t = 2. [2+1]

b) If $\vec{A}$ and $\vec{B}$ are differentiable function of a scalar u, then find [2]

i) $\frac{d}{du}(\vec{A}\cdot\vec{B})$ and

ii) $\frac{d}{du}(\vec{A}\times\vec{B})$

c) Show that magnitude of a vector remains unchanged after coordinate transformation. [2]

d) If coordinate axes (x,y) transform into another coordinate axes $(x';y')$ by the equation $x'=ax+by$, $y'=cx+dy$ where a, b, c, d are constants. prove for orthogonal transformation $a^2+c^2=1$. [3]

3. a) Write stokes' theorem. Prove it from the fundamental idea of curl. [1+4]

b) Show that $\vec{\nabla}\phi$ is a vector perpendicular to the surface $\phi(x,y,z)=C$, where C is a constant. [2]

c) Suppose $\vec{\nabla}\times\vec{A}=0$, Evaluate $\vec{\nabla}\cdot(\vec{A}\times\vec{r})$. [3]

4. a) Let, $\vec{F}=\frac{-y\hat{i}+x\hat{j}}{(x^2+y^2)}$

i) Calculate $\vec{\nabla}\times\vec{F}$.

ii) Evaluate $\oint\vec{F}\cdot d\vec{r}$ around any closed path and explain the results. [2+3]

b) Verify the divergence theorem for $\vec{A}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$. taken over the region bounded by $x^2+y^2=4$, z = 0 and z = 3. [5]

5. a) Find the value of scale factors of cylindrical coordinate system. [2]

b) Solve the differential equation : $(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+6y=0$ around the point x = 0. [4]

c) Express the divergence operator in spherical polar coordinates starting from the expression of the same in Cartesian coordinates. [4]

Unit – II

(*Answer <u>any three</u> questions*) [3×10]

6\. a) i) Prove that in spherical polar coordinates (r, θ, ϕ), the position vector of a moving particle is $\vec{r} = \hat{i} r \sin\theta \cos\phi + \hat{j} r \sin\theta \sin\phi + \hat{k} r \cos\theta$.

ii) Show that the velocity in spherical coordinates is given by $\vec{v} = \hat{r}\,\dot{r} + \hat{\theta} r\dot{\theta} + \hat{\phi} r \sin\theta \dot{\phi}$. [5]

b) A particle moves in an elliptical path given by $\vec{r} = \hat{i} a \cos\omega t + \hat{j} b \sin\omega t$ where a, b, ω are constants.

i) Find the angle between velocity nd acceleration at time $t = \dfrac{\pi}{4\omega}$.

ii) Show that the angular momentum vector about the origin is constant. Can you explain the result? [2+3]

7\. a) Write down the Galilean Transformation (G.T) equation between the space and time. variables of two inertial frames S and S', is uniform relative motion with velocity $\vec{v}$, stating clearly any assumptions you make. [2]

b) i) Show that the relative separation between two moving particles as measured in S and S', respectively, is Galilean invariant. [1]

ii) How does the position vector of the centro-of-mass of the two particles transform under a G.T.? [2]

c) i) State and explain the work-energy theorem for a particle moving in a force field $\vec{F}$. [2]

ii) If $\vec{F}$ is a conservative field in the region under consideration, show that the total mechanical energy is constant. [3]

8\. a) i) A small body of mass m is projected vertically downwards with a speed u in a medium which offers a resistance mkv when the speed is v. Find the distance it falls in time t.

ii) If, T seconds later, an exactly similar body is projected vertically downwards with speed u'. Show that the separation between the bodies approaches a limit given by, $S = (u - u' + gT)/k$. [4]

b) Show that the general equation of motion of a body of variable mass m moving in a force field $\vec{F}$, with instantaneous velocity $\vec{v}$, through a dust stream whose steady velocity is $\vec{u}$, is of the form, $\dfrac{d}{dt}(m\vec{v}) = \vec{F} + \vec{u}\dfrac{dm}{dt}$. [3]

c) A spherical rain drop of instantaneous radius r and mass m falls through a stationary cloud under gravity. The rate of acceleration of mass is proportional to the product of its surface area and instantaneous speed v. Show that the acceleration of the drop as long as it falls through the raincloud is g/7. [Given v = 0, when r = 0] [3]

9\. a) Prove that for a system of particles, the angular momentum about a point is the sum of the angular momentum of a particle of equal mass placed at the centre of mass and the angular momentum of the particles about the centre of mass. [3]

b) Two binary stars of masses 10^{21} kg and 2×10^{21} kg are 10^{6} km apart. They rotate about their centre of mass with angular velocity ω. Find the value of ω. [3]

c) A 6 kg object is moving on a frictionless plane at a speed of 350 ms^{-1}. It explodes into two fragments one of 2 kg and one of 4 kg with velocities along the initial direction of motion of 250 ms^{-1} and 200 ms^{-1} respectively.

i) What is the total momentum before and after explosion? [2]

ii) What is the total momentum before and after the explosion in the centre of mass frame? [2]

Unit – III

(*Answer <u>any one</u> question*) [1×10]

10. a) Consider a pendulum of length ℓ and a bob of mass m at its end moving through oil with θ decreasing. The massive bob undergoes small oscillations, but the oil retards the bob's motion with a resistive force proportional to the speed with $F_{res} = 2m\sqrt{\frac{g}{\ell}}\,\ell\dot{\theta}$. The bob is initially pulled back at t = 0 with $\theta = \alpha$ and $\dot{\theta} = 0$. Find the angular displacement θ and the velocity $\dot{\theta}$ as a function of time. [3]

b) A sinusoidal external force is applied on a damped harmonic oscillator, what will be the transient effect of the oscillator? Find the steady state solution of the oscillator. Sketch the variation of amplitude and phase difference with angular frequency for different dampings. [7]

11. a) A horizontally mounted spring stretches 0·03 m when subjected to a force 6N. A 0·5 kg mass is attached to the free end and given an initial displacement of 0·015 m and an initial velocity of 0·4 ms^{-1}. Find the time period and amplitude of the motion. [2+2]

b) Distinguish between amplitude and velocity resonance. Sketch the relationship between the driving frequency and the amplitude of the driven oscillator for two different damping constants. Hence discuss qualitatively the idea of sharpness of resonance. [2+2+2]

Unit – IV

(*Answer <u>any three</u> questions*) [3×10]

12. a) State Fermat's principle. Show that if a point source is placed at the focus of a paraboloidal mirror the reflected ray is perfectly parallel to axis. [1+2]

b) A light ray is refracted at a curved spherical surface of radius of curvature R. The refractive index of the medium on the incident side is n_1 and that on the other side is n_2. Show by Fermat's principle that $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$ where u, v are object and image distance respectively. [4]

c) Show that a spherical refracting surface is aplamatic with respect to certain positions of the object. Determine these positions. [3]

13. a) A lens of thickness t, is made from a glass of refractive index μ, with R_1 and R_2 as the radii of curvature of two refracting surfaces. Obtain an expression of the system matrix of the above lens. [6]

b) Using the above formulation, deduce the lens maker formula for a thin lens. [4]

14. a) What do you mean by dispersive power of a refracting material? Deduce the condition of achromatism of two thin lenses, of same material, separated by a distance. [1+3]

b) It is desired to make a converging achromatic lens of equivalent focal length 30 cm by using two lenses of material A and B. If the dispersive power of A and B are in the ratio 1 : 2, find the focal length of each lens. [3]

c) Calculate the axial chromatic error of a lens for nonparallel rays. [3]

15. a) What is spherical aberration? State any two methods for reducing spherical aberration. [3]

b) Draw a neat diagram to show astigmatism in a lens. Show sagittal plane and meridional plane in your diagram. [3]

c) Calculate and show in a diagram the positions of principal points and the focal points of a Ramselen's eyepiece. [4]

16. a) Deduce the Helmholtz-Lagrange relation for refractions at spherical surfaces. [4]

b) What is apochromatic optical system? [2]

c) Find an expression for the achromatic condition for two thin lenses separated by a distance. [4]

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